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Interoffice Correspondence

Date:	August 7, 2008
To:	Shane McDonald (KOP), Scott Thompson (WHI)
From:	Ed Garvey, Juliana Atmadja & John Cole, NNJ; John Kern (Kern Statistical Services)
Re:	Bathymetry Block Analysis
Project No.:	4553054

This memo describes the results of the statistical methods used to analyze the erosional and depositional area of the Lower Passaic River based on the available bathymetric surveys.

Two statistical methods (unpaired T-test and analysis of covariance) were used to evaluate the erosional and depositional area of the Lower Passaic River based on the 1995 and 1996 bathymetric surveys. The analytical methodologies are described in the attached memorandum from John Kern. For the smaller grid sizes (30 x 300 ft and 40 x 400 ft), Method II results in a larger total significant area of change when compared to Method I (Table 1). This is because Method II takes into account the spatial variation among the bathymetric measurements. The biggest size (100 x 1000 ft) selected for Method II resulted in the smallest total significant area of change. This grid size can be viewed as the upper bound of the grid cell size. In addition, the net depositional volume calculated from Method II is in close agreement with the results obtained by the methods presented in the TIN-based analysis from the Comprehensive Conceptual Site Model (CSM). Therefore, Method II was used to analyze the bathymetric changes for all historical TSI and USACE bathymetric surveys performed in 1995, 1996, 1997, 1999, 2001, and 2004. Bathymetric changes for the following year intervals were examined:

- 1995 to 1996
- 1996 to 1997
- 1997 to 1999
- 1999 to 2001
- 2001 to 2004
- Sum of years 1995 to 2004

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For each year interval comparison, four different grid-sizes were selected, as follows:

- 30 x 300 ft
- 40 x 400 ft
- 50 x 500 ft
- 100 x 1000 ft

Table 2 shows the results of Method II statistical analysis. For each of the comparisons, about two-thirds of the river bottom area shows a statistically significant change from year to year. The ratio of overall (1995-2004) significant area to total area ranges from 0.61 to 0.69. The average net depositional thickness was also calculated for each scenario. The overall average net deposition (1995 to 2004) from Method II ranges from 8.4 to 9.3 inches (Table 2). This is comparable to the values obtained by both the TIN-based and simple point-based mean bathymetric comparisons presented in the CSM, which estimated average net depositional thicknesses of 8.9 and 9.8 inches, respectively (Table 2). The individual year interval average net depositional thickness also agrees well with the TIN-based bathymetric analysis (see yellow highlights in Tables 2 and 3).

The significant net depositional volume was also calculated using Method II and the results are in close agreement with the TIN-based analysis. The overall net depositional volume between 1995 and 2004 is about 400,000 cy (see Tables 2 and 3). In addition, the ratio of erosional to gross depositional volume was calculated. This ratio reflects the amount of resuspended sediment relative to the gross accumulation of sediment on the river bottom occurring between the bathymetric surveys. It can be thought of as reflecting the average fraction of deposited sediments originating as resuspended sediment. The mean ratio of erosional to gross depositional volume based on Method II is about 0.45. This ratio closely agrees with the ratio calculated in the CSM using the TIN-based analysis, a value of 0.55. These results are also in close agreement with the prediction of the fraction of resuspended solids from the EMB, and further confirm this important conclusion from the EMB report.

One final observation can be made concerning the small, but generally consistent, difference between the TIN-based results and those of Method II. In general, the Method II results show smaller absolute volumes of erosion and gross deposition relative to the TIN results. The Method II results also yield a slightly lower erosion - deposition ratio. This might be expected, given the differences in the two approaches. Specifically, the TIN-basis comparison applies an absolute threshold (3.75 in) to identify a difference between bathymetric surveys but makes no requirement as to the clustering of the observations above the threshold. In this manner, all differences of sufficient magnitude are counted in the total, regardless of their spatial distribution. In contrast, Method II makes no specific absolute magnitude requirement but does require a sufficiently large grouping of points (*e.g.*, about 9 points for the 30x300 grid cell) in order to generate sufficient statistical power. This gathering of points is expected to reduce some of the sensitivity to local changes, essentially by averaging. As a result, Method II yields slightly smaller volumes in most comparisons. In preparing this analysis, however, Malcolm Pirnie, Inc and Kern Statistical Associates have been able to largely minimize the effects of the averaging process, thus yielding comparable estimates of sediment deposition and erosion by both methods.

Attachment: Memo from John Kern (Kern Statistical Services) on Statistical Methods to Detect Change in Nearly Paired Bathymetric Surveys.

Table 1
Comparison between Method I (Unpaired T-Test) and Method II (Analysis of Covariance) Statistical Analyses on Bathymetry Change

Year Interval	Grid Dimension	Gridsize (ft^2)	Gridsize (acres)	Total # of Cells		Norm Significant Erosional Area (acres)		Norm Significant Depositional Area (acres)		Norm Total Significant Area (acres)		Significant Area/Total Area (%)		Avg Loss in Erosional Cells (in)		Avg Gain in Depositional Cells (in)		Avg NET Depositional Thickness (in) over Entire River (350 acres)		Norm Significant Erosional Vol (yd^3)		Norm Significant Depositional Vol (yd^3)		Norm Significant NET Depositional Vol (yd^3)		E/D (Erosion/deposition)	
	Std Area (acres) =	350		Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II	Method I	Method II
1995-1996	30X300	9,000	0.21	2,077	1,698	4.7	39	78	160	83	200	24%	57%	12	5.1	12	8.6	11	3.4	7,400	27,100	124,100	185,000	117,000	158,000	0.06	0.15
	40x400	16,000	0.37	1,229	1,048	4.3	51	75	190	79	241	23%	69%	9	4.5	12	8.1	10.5	3.7	5,200	31,000	117,800	206,000	113,000	175,000	0.04	0.15
	50x500	25,000	0.57	813	767	5.6	44	80	181	85	225	24%	64%	8.5	4.6	11	7.7	10	3.4	6,400	27,200	122,100	186,000	116,000	159,000	0.05	0.15
	100x1000	100,000	2.3	238	226	7.4	46	87	81	94	127	27%	36%	5.1	3.8	9.8	5.8	8.6	0.8	5,000	23,600	114,200	63,000	109,000	39,600	0.04	0.37

Table 2
Bathymetry Change Results using Method II (Analysis of Covariance)

Year Interval	Grid Dimension	Gridsize (ft^2)	Gridsize (acres)	Total # of Cells	Norm Significant Erosional Area (acres)	Norm Significant Depositional Area (acres)	Norm Total Significant Area (acres)	Significant Area/Total Area (%)	Avg Loss in Erosional Cells (in)	Avg Gain in Depositional Cells (in)	Avg NET Depositional Thickness (in) over Entire River (350 acres)	Norm Significant Erosional Vol (yd^3)	Norm Significant Depositional Vol (yd^3)	Norm Significant NET Depositional Vol (yd^3)	E/D (Erosion/deposition)
	Std Area (acres) =	350													
1995-1996	30X300	9,000	0.21	1,698	39	160	200	57%	5.1	8.6	3.4	27,100	185,000	158,000	0.15
	40x400	16,000	0.37	1,048	51	190	241	69%	4.5	8.1	3.7	31,000	206,000	175,000	0.15
	50x500	25,000	0.57	767	44	181	225	64%	4.6	7.7	3.4	27,200	186,000	159,000	0.15
	100x1000	100,000	2.3	226	46	81	127	36%	3.8	5.8	0.8	23,600	63,000	39,600	0.37
1996-1997	30X300	9,000	0.21	1,686	138	64	201	57%	4.5	5.6	-0.75	83,100	47,600	-35,500	1.7
	40x400	16,000	0.37	1,090	162	75	237	68%	4.4	5.8	-0.80	96,000	58,200	-37,900	1.7
	50x500	25,000	0.57	672	173	80	253	72%	4.2	5.5	-0.81	97,300	59,000	-38,200	1.6
	100x1000	100,000	2.3	224	83	33	116	33%	6.2	9.0	-0.62	68,900	39,900	-28,900	1.7
1997-1999	30X300	9,000	0.21	1,443	37	173	210	60%	7.4	6.5	2.4	37,200	149,800	112,600	0.25
	40x400	16,000	0.37	947	54	187	240	69%	6.1	6.3	2.4	44,200	159,000	114,800	0.28
	50x500	25,000	0.57	650	48	186	235	67%	5.9	6.6	2.7	38,200	166,200	128,000	0.23
	100x1000	100,000	2.3	192	40	102	142	41%	6.6	4.6	0.6	35,300	63,200	27,900	0.56
1999-2001	30X300	9,000	0.21	1,468	63	155	219	63%	6.1	7.2	2.1	52,200	149,500	97,300	0.35
	40x400	16,000	0.37	975	65	169	234	67%	5.6	8.0	2.8	49,300	181,800	132,500	0.21
	50x500	25,000	0.57	663	56	172	228	65%	5.7	7.7	2.9	43,000	178,200	135,100	0.19
	100x1000	100,000	2.3	198	39	99	138	39%	6.6	4.6	0.6	34,300	61,300	27,100	0.36
2001-2004	30X300	9,000	0.21	1,408	106	125	230	66%	5.8	9.1	1.5	82,600	151,900	69,200	0.54
	40x400	16,000	0.37	932	127	126	252	72%	6.0	9.2	1.1	101,900	154,900	53,000	0.66
	50x500	25,000	0.57	648	129	124	253	72%	6.4	8.8	0.77	111,100	147,100	36,000	0.76
	100x1000	100,000	2.3	193	78	60	138	39%	6.1	3.1	-0.84	64,400	24,800	-39,600	2.60
Sum	30X300	9,000	0.21	1,541			212	61%			8.5	282,200	683,800	401,600	0.41
1995-2004	40x400	16,000	0.37	998			241	69%			9.3	322,400	759,900	437,400	0.42
	50x500	25,000	0.57	680			239	68%			8.9	316,800	736,500	419,900	0.43
	100x1000	100,000	2.3	207			132	38%			0.6	226,500	252,200	26,100	0.90

Notes:
1. #N/A - Not Available
2. Yellow highlights indicate close agreement with the TIN basis analysis results.
3. Blue highlights indicate close agreement with the point basis analysis results.

Table 3
Based on Volumes from Table 11-10 and from Point Basis Table
- Grid (TIN) Basis Excludes Neutral Volume in Calculation

Comparison	Erosional (cy)	Depositional (cy)	Neutral (cy)	Net (cy)	e/d	Average Change in Depth (in)
TIN Basis 1995_1996	60,416	217,303	6,292	156,887	0.278	3.48
Total Point Avg 1996_1995				112,962		2.51
TIN Basis 1996_1997	109,503	73,899	9,418	-35,604	1.482	-0.79
Total Point Avg 1996_1997				-60,312		-1.34
TIN Basis 1997_1999	69,956	189,857	10,527	119,900	0.368	2.66
Total Point Avg 1997_1999				155,896		3.46
TIN Basis 1999_2001	81,312	211,918	7,512	130,606	0.384	2.90
Total Point Avg 1999_2001				198,953		4.42
TIN Basis 2001_2004	150,977	166,296	3,050	15,319	0.908	0.34
Total Point Avg 2001_2004				34,341		0.76
TIN Basis 1995-2004	472,164	859,272	36,798	398,972	0.549	8.9
Total Point Avg 1995-2004				441,839		9.8

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Statistical Methods to Detect Change in Nearly Paired Bathymetric Surveys

Introduction

Bathymetry data have been collected over a period of years along the Lower Passaic River. The nature of changes in bathymetric surfaces over time has been studied, since erosion and deposition influence decisions regarding selection of remedial alternatives. In particular, it has been hypothesized that in some areas both erosion and deposition have occurred in different time periods. Bathymetric data have been collected using both single beam and multi-beam technologies. When single beam surveys were conducted, attempts were made to measure bathymetry at the same locations as in previous years, but it is generally not possible to precisely re-occupy specific locations in successive years. Transects tended to fall within approximately 18 meters of previous transects, so data could be considered to be nearly or "quasi" paired. Analyses conducted to date have attempted to pair observations by using interpolation, followed by comparison of interpolated surfaces, or by ad-hoc pairing of observations that are judged to be "close" together. Results of these analyses have not been completely consistent and have also not included completely satisfying uncertainty analyses. In response to critical reviews of these previous analyses, alternate methods are proposed in this memo for estimating change in bathymetric surfaces, with rigorous statistical uncertainty estimates.

Methods

Because sample data could not be rigorously paired by sampling design and because ad-hoc procedures for pairing data have stimulated discussion of its potential adverse effects on the estimation procedures, the approach developed here avoids pairing altogether, relying instead on grouping data into rectangular cells or "boxes" within which mean elevations can be compared. The idea is to balance the resolution of the comparisons against the number of sample elevations needed to detect differences in mean elevation among years. In general, statistical power to detect differences increases with number of samples per comparison. However, increasing the number of samples per comparison in this framework results in an increase in the size of the area within which data are aggregated. This aggregation has the potential mitigating effect of reducing the difference in mean elevation due to confounding of temporal and spatial variation.

Two procedures were implemented to test for temporal differences in bathymetric elevation within specified boxes. Method I was based on the two sample T-test with Satterthwaite approximation to adjust for potentially unequal variances (Satterthwaite, 1946 and Welch 1947). The second method incorporates the spatial coordinates into the analysis so that spatial variation in the bathymetric surface can be accounted for. When the bathymetric surface varies substantially, this second method is expected to provide higher power to detect differences than the un-paired T-test. The general approach is to fit parallel polynomial regression surfaces to the bathymetric elevations for each year, within each box, and then test for a vertical shift in these parallel surfaces.

Method I (Unpaired T-Test)

The unpaired two sample Student's T-test is conducted as follows. Let $z_{11}, z_{12}, \dots, z_{1n1}$ and $z_{21}, z_{22}, \dots, z_{2n2}$ represent two separate unpaired samples from years 1 and 2 within a particular grid cell of interest. The means of the populations from which these samples were collected are compared by calculating sample means and variances for each year and constructing the T-statistic $T = (\bar{z}_1 - \bar{z}_2)/\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ which is compared to a Student's $T_{\alpha;\, df}$ statistic at the α level of significance and df degrees of freedom calculated using Satterthwaitte's equation:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^4}{(n_1 - 1)n_1^2} + \frac{s_2^4}{(n_2 - 1)n_2^2}\right)}$$

Method I assumes that the bathymetric surface in each box can be represented (modeled) as a horizontal surface with individual measurements fluctuating around this constant mean elevation. For relatively small grid cells this is a reasonable model, but for larger cell sizes it is more reasonable to expect spatial trends in the bathymetry, which reflect the general geomorphic features of the river. Particularly with bathymetry transects spaced tens to hundreds of feet apart it is likely that the true bathymetry may not be well approximated by the horizontal surfaces assumed by the two sample T-test. This "mis-specification" of the model for the population mean can be expected to reduce its power to detect differences because variation of the true population mean surface and the horizontal surface is effectively lumped with the sample variance, in effect overstating the level of uncertainty in the estimated difference in mean values.

Method II (Analysis of Covariance)

A standard statistical approach for handling the shortcomings of Method I is known as analysis of covariance (ANCOVA; Neter et al 1990). This approach is common in human health applications where it is well known that simple comparisons of things like cancer incidence among groups of people is inappropriate without consideration of the effects of age or other factors. The ANCOVA approach effectively allows comparisons to be made between subjects of like age and other ancillary variables.

This is often called controlling for ancillary factors. Effectively it is like pairing subjects based on the distributions of ancillary factors. In the context of comparing unpaired bathymetry data, controlling for spatial variation represents a statistically valid way to approximately pair comparisons in a statistically rigorous way.

Define $z_1(x,y)$, and $z_2(x,y)$ to be the bathymetric measurements in years one and two respectively with their associated geographic coordinates x and y. Rather than assuming that the mean within each box is constant, assume that the mean can be better approximated with a moderately flexible polynomial surface. In particular, a third order polynomial in x and y is proposed

$$z_{ik}(x_{ik},y_{ik}) = \beta_0 + \beta_1 x_{ik} + \beta_2 y_{ik} + \beta_3 x_{ik}^2 + \beta_4 y_{ik}^2 + \beta_5 x_{ik} y_{ik+} + \beta_6 x_{ik} y_{ik}^2 + \beta_7 x_{ik}^2 y_{ik} + \beta_8 x_{ik}^3 + \beta_9 y_{ik}^3 + \beta_{10} I_{ik}(year)$$

where i=1,2,3…n_k indexes the sampling locations for year k, and $I_k(year)$ is assigned a 0 for the first year under consideration and a 1 for the second year under consideration. The test for a shift in bathymetric elevation among years within the grid cell of interest is then a test of the null hypothesis $H_0: \beta_{10} = 0.0$. This hypothesis is rejected when $|t| = \left|\frac{(\hat{\beta}_{10})}{se(\hat{\beta}_{10})}\right| > T_{\alpha/2;\, df=n_1+n_2-11}$, where this ratio is distributed as a Student's T statistic with (n_1+n_2-11) degrees of freedom. Following is a short example illustrating these two procedures.

Example

A bathymetric surface was simulated using a geostatistical simulation algorithm providing an array of spatially correlated observations suitable to test the two procedures. A second map of bathymetry was simulated by by adding a randomly distributed value to each location to shift the values from the first map. These random values had mean 1.0 feet and variance 0.25 feet^2. This resulted in two bathymetry surfaces with the first 1.0 foot higher on average, although not all locations were shifted uniformly. This is similar to what could be expected in practice. The simulated maps can be seen in Figure 1. To illustrate the analysis, a single 40 foot by 200 foot block area was selected and then each map was randomly sampled at 12 and 15 locations, respectively.

Figure 2 provides perspective views of the two simulated bathymetric surfaces as well as the fitted third order polynomials. Sample data are plotted as well to provide a comparison between the fitted model and the simulated sample bathymetric measurements.

The results of the Satterthwaite's T-test are summarized in Table 2. The estimated difference in mean elevation was 1.12 feet with a standard error of 0.19 feet. Based on this procedure one would conclude that the likely hood of observing these data under the null hypothesis of no change (up or down) in elevation would be approximately three in one million (3.03e-6).

The results of the ANCOVA approach are summarized in Table 3 and the estimated difference among years, as given by the estimated regression coefficient for the year, was 1.0004 feet with standard error 0.106 feet. It can be seen that the regression approach estimated the true 1 foot difference more accurately. In addition, this estimate was more precise, as exhibited by the standard errors. The adjusted R-squared for the model was 91%.

An assumption of the regression analysis is that the residual errors are statistically independent. Because the sample was from a spatially autocorrelated process, it is expected that the residual errors would be spatially correlated, therefore violating the independence assumption. It is often assumed by practitioners that design-based methods such as regression are not valid due to the autocorrelation in the underlying process. However, often when the mean surface is parameterized as a moderately flexible regression surface, the resultant residual errors are uncorrelated. In this case, Moran's I (Moran, 1950) was used to test for spatial correlation in the regression residuals and whatever spatial information remained in the errors was not adequate to reject the null hypothesis of independent errors (I=-0.09,se=0.07; p=0.15). This suggests that the spatial information was captured adequately by the third order polynomial and that any other systematic variation in the simulated data was insufficient to warrant the additional complexity of modeling the spatial correlation structure.

Discussion

The methods described in this memo are drawn from standard statistical procedures for estimating differences among means. The regression method has the advantage of utilizing more of the readily available information than the Satterthwaite procedure and also provides the benefit of handling the unpaired samples in a quasi-paired fashion, without the need for ad-hoc decisions regarding how to pair measurements. In the example, the results of both analyses are consistent in that a one foot change in elevation is easily detected using either method. However, the regression approach has the added value of estimating the difference more accurately and precisely. This can be expected to result in greater power to detect subtle changes in bathymetry and better quantification of the time varying deposition and erosion patterns that appear to be prevalent in the Lower Passaic River.

It is anticipated that some may suggest that these procedures are inferior to other interpolation based approaches, such as kriging. This potential objection has been thoroughly considered and it important to note that both the T-test and the regression approach can be described as general linear models of the form $Z = X\beta + \varepsilon$ where $X\beta$ represents the mean surface and ε represents a mean zero residual process. If kriging were used to interpolate the surfaces prior to subtraction, one would assume that the residual process is spatially correlated and then proceed to calculate differences of kriged cell values. The end result would be an estimated difference based on the weighted average of sample values. Because the sampling design is based on regularly spaced transects with regularly spaced samples along transects, the resulting sample weights would be nearly equal. Interestingly, the Satterthwaite estimate is the difference of unweighted means, much like one would produce with kriging. Therefore, very similar estimated differences are expected from either procedure, because of the mathematical similarity of the estimates.

In general, one may be concerned by the failure of these procedures to account for spatial correlation in the residual errors. In this example the response surface was adequate to filter out the spatial information. More generally, because the sample locations are not spatially clustered, spatially weighted or unweighted estimates of the difference in means are unbiased. In theory, estimators that incorporate (correctly) the spatial correlation in the residual errors are expected to result in higher power tests for differences. In practice, the theoretical advantages may not be realized. This issue is discussed by Papritz and Webster (1995a and b) and it was their finding that uncertainty in variogram parameters may swamp the possible theoretical gains in power. As a result they concluded that models based on design-based procedures (i.e. regression models, or stratified sampling) may be equally, or more, powerful in practice. Additionally, the Method II procedure has the advantage of explaining much of the spatial variability in the residual errors through estimation of a flexible mean surface. Residual errors in this example were found to be uncorrelated. In general this would be expected to hold true for all but the most complex surfaces and irregular sampling designs.

Table 1. Example data illustrating Satterthwaite's and ANCOVA procedures.

Year	I(year)	x(feet)	y(feet)	Elevation z(x,y)	i
1995	0	-149.83	134.59	-0.91	1
1995	0	-142.99	143.61	-0.61	2
1995	0	-139.97	1.00	-0.74	3
1995	0	-140.12	163.44	-0.24	4
1995	0	-112.78	230.53	0.35	5
1995	0	-148.90	156.95	-0.90	6
1995	0	-114.06	3.07	-0.99	7
1995	0	-139.39	90.20	-1.08	8
1995	0	-112.23	135.47	-0.02	9
1995	0	-126.53	168.54	-0.42	10
1995	0	-139.65	58.98	-1.05	11
1995	0	-130.87	49.01	-0.97	12
1996	1	-146.92	144.43	0.07	1
1996	1	-107.92	183.37	1.15	2
1996	1	-141.03	224.77	1.10	3
1996	1	-123.95	50.62	-0.24	4
1996	1	-117.08	152.75	0.55	5
1996	1	-110.16	5.96	0.08	6
1996	1	-126.60	119.52	0.32	7
1996	1	-134.25	160.80	1.16	8
1996	1	-122.20	72.29	-0.17	9
1996	1	-131.55	239.31	0.91	10
1996	1	-115.23	82.24	-0.03	11
1996	1	-138.92	242.18	0.96	12
1996	1	-128.97	159.37	1.02	13
1996	1	-132.26	55.13	0.42	14
1996	1	-120.06	87.46	0.09	15

Table 2. Summary of Satterthwaite's T test.

year	n	mean	variance	|diff|	SE	|T|	df	Prob(T>|t|)
1995	12	-0.63209	0.21059	1.12353	0.187785	5.983069	24.6667	3.03E-06
1996	15	0.49144	0.26571					

Table 3. Summary of regression parameter estimates for third order polynomial surface fit to bathymetric elevations.

Parmameter	Estimate	Standard Error	T	Prob(T>|t|)
Intercept	0.460200	0.088333	5.21	0.0001
x	-0.009328	0.010286	- 0.91	0.3779
y	0.009608	0.001665	5.77	**0.0000**
x^2	-0.000658	0.000411	- 1.60	0.1291
y^2	0.000013	0.000010	1.30	0.2115
x*y	0.000045	0.000083	0.54	0.5935
x*y^2	0.000074	0.000031	2.41	**0.0283**
x^2*y	0.000000	0.000000	- 3.04	**0.0078**
x^3	0.000000	0.000001	- 0.22	0.8256
y^3	0.000003	0.000008	0.44	0.6681
Year (df=16)	1.004580	0.105881	- 9.49	**5.67943E-08**

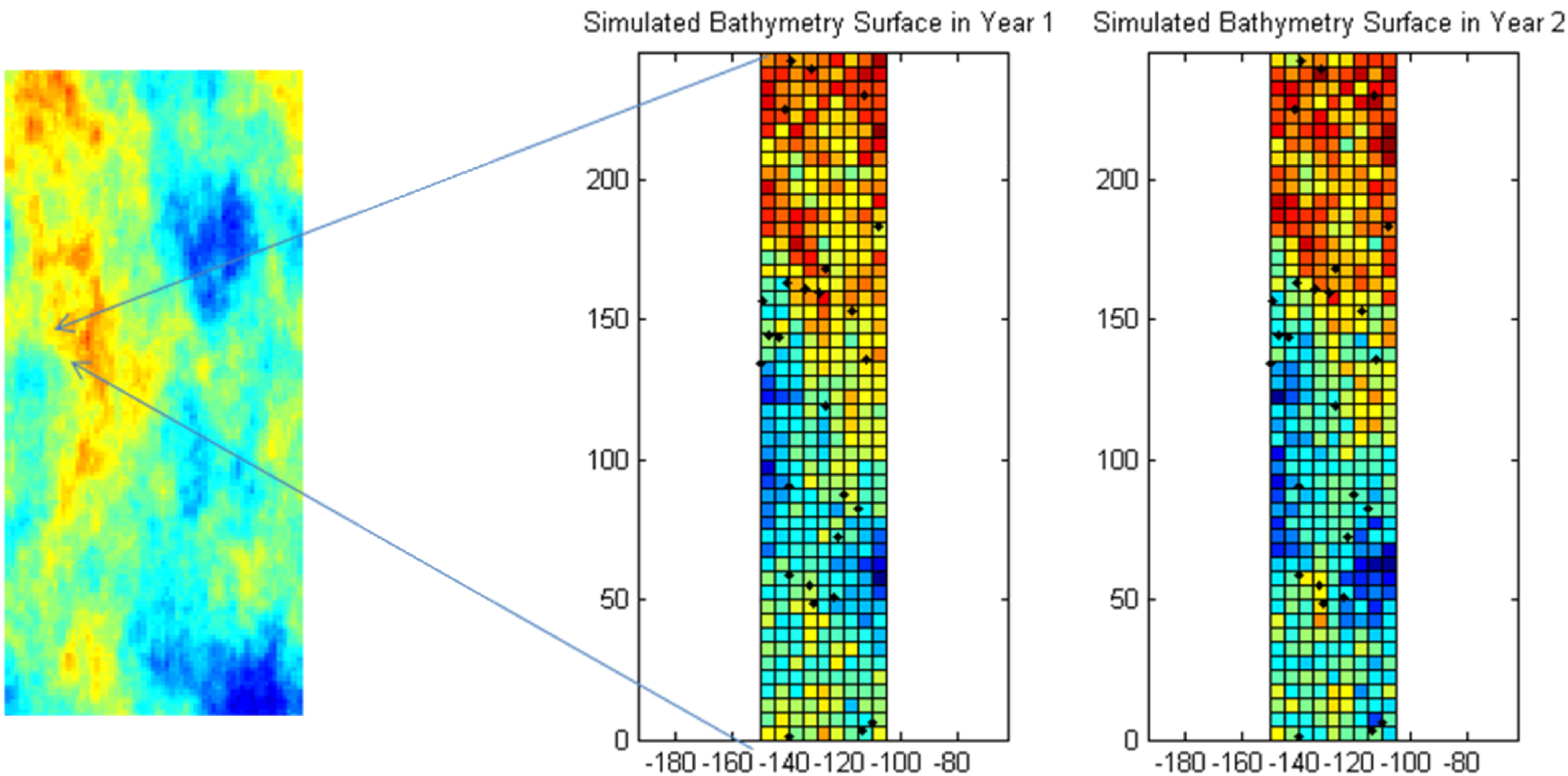

Figure 1. Simulated bathymetric surface with an average shift of 0.5 feet from year 1 to year 2. The 0.5 foot shift was applied by adding a normally distributed random variable with mean 0.5, variance 0.25 to each simulated bathymetry value. The test is applied to a 40 by 200 foot block of locations. Black dots represent sampling locations.

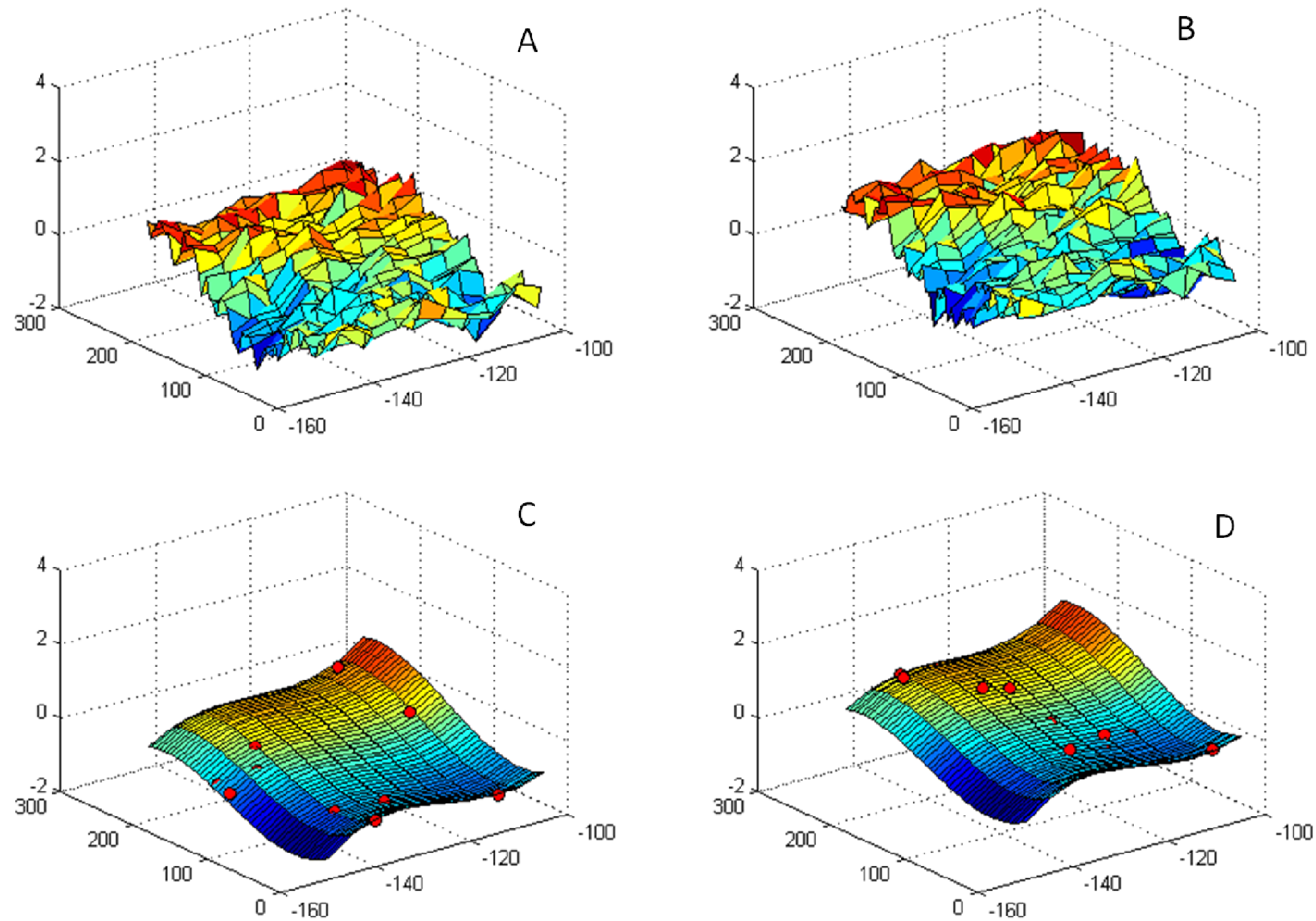

Figure 2. Simulated bathymetry surfaces for year one (Panel A) and year two (Panel B). Estimated third order polynomial surfaces for year one (Panel C) and year d (Panel D). Red circles represent sample values in comparison with the fitted surfaces..

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